

Deterministic Chaos in Quantum Field Theory

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We discuss the necessity and demonstrate the validity of introduction the notion of deterministic chaos in quantum field theory. Brief review of the existing approaches to this problem is given. We compare proposed chaos criterion for quantum fields with existing ones. Its consequences in particle physics are also discussed.

§1. Introduction

Chaos phenomenon attracts much attention in various fields of natural sciences, from celestial mechanics¹⁾ (pp.349-359) and behavior of social systems²⁾ to quantum mechanics³⁾ and atomic physics.⁴⁾ Though the understanding of the reasons and regimes of chaotic behavior in classical (Hamiltonian) systems is already achieved in main,⁵⁾ problems connected with properties, manifestations and even definition of deterministic chaos in quantum world are still basically open.^{6),7)} Prevalence of chaotic motions in the majority of natural phenomena explains the fundamental and applied significance of its study. Particulary, chaotic behavior is intrinsical to classical gauge fields in modern theories of particle interactions^{8),9),10)} as well it plays crucial role for quantum tunneling control, which is of practical importance.¹¹⁾

In this paper we discuss chaos phenomenon in the context of quantum field theory (QFT). We premise the consideration of our main topic by some motivations. Investigation of the classical gauge field dynamics from the viewpoint of chaos was started in a few years after recognition of gauge theories based on $SU(3)$ and $SU(2) \otimes U(1)$ groups as theories of strong and electroweak interactions respectively. Nonabelian gauge fields form complicated nonlinear system with large number of degrees of freedom, which behavior demonstrates a lot of regimes when parameters vary. Besides the investigation of classical gauge fields to be interesting from the viewpoint of nonlinear dynamics, it is also important from the viewpoint of particle physics (particulary QCD), where large progress is achieved in understanding of perturbative phenomena, but comparatively little is known about the non-perturbative evolution of quarks and gluons. Namely, the confinement phenomenon responsible for absence of free colored objects is waiting for its explanation (however a lot of results is obtained in this direction, for review see Ref.¹²⁾). The problem is that there are no exact methods for description of non-perturbative evolution of quantum fields, when coupling between them is not too small (there is no small parameter) and perturbative expansions can not be applied. The first step for understanding of quantum dynamics in this coupling regime is the investigation of the classical behav-

ior of fields in this region. However, even analysis of classical dynamics appears to be a hard problem, because of infinite number of degrees of freedom of field system. Its reduction to finite one is needed in order existing methods to be applied. The easiest way is to consider model field system with small number of freedoms. It is achieved by imposing "by hands" some additional restrictions on the behavior of fields. For example it can be the condition of spatial homogeneity of gauge fields, which was used by many authors ^{*)}, for instance^{8),10),14)} (see references therein). On the examples of spatially homogeneous model field systems it was demonstrated that the behavior of nonabelian classical gauge fields is inherently chaotic.⁸⁾ Existing numerical methods let to consider more realistic model of continuous in space-time gauge field system possessing thousands of degrees of freedom, namely, gauge fields on a lattice. Maximal Lyapunov exponent¹⁵⁾ and total Lyapunov spectrum¹⁶⁾ were numerically determined. Lattice simulations straightly demonstrated that the dynamics of gauge fields on a lattice is chaotic. Thus one can expect that real classical gauge field system demonstrates chaotic behavior, however some differences between gauge field system on a lattice and in continuous space-time have been noted.¹⁷⁾

Chaotic behavior of classical gauge fields does not directly indicate how should chaos phenomenon become apparent in quantum case and what is deterministic chaos in quantum field theory. However there are evidences of chaotic behavior in modern quantum field theories such as QCD, namely signs of chaos in branching processes.¹⁸⁾ This approach do not refer to the chaotic behavior of classical gauge fields and thus represents alternative way to the notion of chaos in quantum field theory. The question about its relation with chaotic dynamics of classical gauge fields is open.

The aim of this work is to present our view on the main problems lying on the path leading to the notion of chaos in quantum field theory and to present our approach to this problem. We consider proposed by us chaos criterion for quantum fields⁷⁾ in the context of existing results obtained in this direction. Also we discuss possible manifestations of deterministic chaos in quantum field theory. Particulary in connection with confinement phenomenon.

The paper is organized as follows. In the Sec.2 we review existing quantum chaos criteria from the viewpoint of their applicability to quantum field theory. Also we discuss our chaos criterion for quantum fields. Its correspondence with the notion of chaos for classical fields is qualitatively demonstrated. In the Sec.3 we provide further justification of proposed chaos criterion. Its correspondence with existing quantum chaos criteria and some consequences are discussed also. Conclusion can be found in the Sec.4.

§2. Chaos criterion for quantum fields

Necessity of probabilistic (or statistical) description of physical systems was primarily realized when behavior of the systems with extremely large number of degrees

^{*)} In the certain Lorenz frame there exist exact spatially homogeneous solutions of gauge field equations of motion. They were primarily obtained in the Ref.¹³⁾

of freedom was studied (statistical mechanics). Mainly in the framework of classical mechanics of Hamiltonian systems it was understood that chaotic behavior of mechanical systems is determined by its local instability rather than a large number of degrees of freedom.¹⁾ Chaotic behavior is prevalent in dynamical regimes of classical mechanical systems. Regular motion is rather exceptional.³⁾ It was noticed in the Ref.⁶⁾ that since classical mechanics is a limiting case of more fundamental quantum mechanics then basic reasons for classical chaotic motion one has to search in quantum world. Exponential divergence of neighbor trajectories (local instability), mixing and other attributes of classical chaos have to be considered as consequences of quantum picture. At the moment the opposite approach dominates. One considers systems with classically regular, mixed or pure chaotic motion and compares properties of their quantum analogues in order to find some differences and special features which are refereed as "quantum chaos". Large progress is achieved in this direction. Particular, universal classes of spectral fluctuations for quantum systems with chaotic behavior in the classical limit have been obtained.¹⁹⁾ Any other approach to the problem of quantum chaos has to be in agreement with these results.

Spectral properties of classically regular or chaotic quantum systems were explained by Gutzwiller's periodic orbit theory.²⁰⁾ The trace formulae²⁰⁾ relates quantum mechanical Green function with the spectral density of states. It provides the bridge between the formulation of quantum chaos in spectral terms and its path integral formulation, which is more convenient for the extrapolation to quantum fields.

For the role of "fundamental" chaos criterion valid for quantum system the value of the system's symmetry violation was proposed.⁶⁾ The parameter quantitatively characterizing symmetry violation for Hamiltonian systems was introduced and correspondence with classical chaos criteria was checked on the examples of Hennon-Heiles system and diamagnetic Kepler problem.^{6),21)} The language of symmetry is the universal language of quantum (not only) physics. However, realization of symmetric approach, proposed in^{6),21)} and well justified in quantum mechanics and nuclear physics, from our point of view, can not be extrapolated in straightforward way for quantum fields. The reason is the absence of evident relativistic covariance and emphasized role of the energy. But the principle of using the symmetry of the system as the measure of its regularity proposed in the Ref.⁶⁾ is fundamental and, undoubtedly, it will be claimed in some form in quantum field theory.

The notion of chaos in quantum field theory can also be introduced in terms of eigenvalue spectrum of the lattice Dirac operator.²²⁾ Particular, it was demonstrated that the nearest-neighbor spacing distribution for the eigenvalue spectrum of the staggered Dirac matrix in quenched QCD on a lattice agrees with the Wigner surmise of random matrix theory.²³⁾

Quantum chaos criterion formulated in classical terms was proposed in the Ref.,²⁴⁾ where quantum system was defined as chaotic if its renormalized action provides *classically* chaotic dynamics (when minimal action principle is applied). The advantages of this approach are the obvious correspondence with classical chaos criteria (if Plank constant equals zero then renormalized action turns into classical action) and relativistic invariance when this criterion is extrapolated to quantum

field theory.

We proposed to give axiomatic formulation of relativistic and gauge ^{*)} invariant chaos criterion in quantum field terms only.⁷⁾ No doubt that some classical motivations which brought us to the formulation of the chaos criterion had to be used. Further justification of the proposed criterion and the check of its correspondence with the notion of classical chaos are to be done.

Now we give some qualitative arguments which bring us to formulation of chaos criterion in quantum field theory.⁷⁾ From statistical mechanics and ergodic theory it is known that chaos in classical systems is a consequence of the property of mixing.⁵⁾ Mixing means rapid (exponential) decrease of correlation function with time.¹⁾ In other words, if correlation function exponentially decreases then the corresponding motion is chaotic, if it oscillates or is constant then the motion is regular.²⁾ We expand criterion of this type for quantum field systems. All stated below remains valid for quantum mechanics, since mathematical description via path integrals is the same.

For quantum field systems the analogue of classical correlation function is the two-point connected Green function

$$G_{ik}(x, y) = -\frac{\delta^2 W[\vec{J}]}{\delta J_i(x) \delta J_k(y)} |_{\vec{J}=0}. \quad (2.1)$$

Here $W[\vec{J}]$ is the generating functional of connected Green functions, \vec{J} are the sources of the fields, x, y are 4-vectors of space-time coordinates.

Thus we formulate chaos criterion for quantum field theory in the following form:⁷⁾

- a) If two-point Green function (2.1) exponentially (or faster) goes to zero when the distance between its arguments goes to infinity then system is chaotic.
- b) If it oscillates and/or slower then exponentially goes to zero in this limit then we have regular behavior of quantum system.

Obviously that proposed chaos criterion is relativistic and gauge invariant and formulated in terms of quantum field theory only. Also it has direct physical sense, namely, if the propagator decays exponentially or faster, then this case corresponds the chaotic behavior of quantum field system. In opposite case the dynamics is regular. Particulary it is seen that dynamics of free quantum fields is always regular as it should be **).

Our aim is to demonstrate the correspondence between proposed definition of chaos for quantum fields and chaotic behavior of classical fields in the semiclassical limit $\hbar \rightarrow 0$. Providing it we justify our quantum chaos criterion initially postulated.

Primarily, we check the agreement between classical criterion of local instability and formulated quantum chaos criterion in framework of quantum mechanics. After this we sketch the prove for QFT. Particulary, we qualitatively justify the correspondence between the quantum chaos criterion and the notion of chaos for classical fields.

^{*)} The reason is that all modern theories of particle interactions are gauge theories.

^{**)} Decreasing of the free field propagator is slower then exponential.

For non-relativistic quantum mechanics the problem is still too complicated to be solved analytically in the closed form. Recent results concerning the calculation of the propagator and Green functions in this case see in Ref.²⁵⁾ We consider the classical systems with Hamiltonian having the form:

$$H = \frac{1}{2}\vec{p}^2 + V(\vec{q}) \quad , \quad \vec{p} = (p_1, \dots, p_N) ; \quad \vec{q} = (q_1, \dots, q_N), \quad N > 1 \quad (2.2)$$

Here N is the (arbitrary) finite number of degrees of freedom.

We consider the case of constant eigenvalues of the classical stability matrix (matrix form of the Jacobi-Hill operator, controlling the linear stability around the classical orbit). The case of non-constant eigenvalues will be qualitatively considered below. Stability matrix for the Hamiltonian (2.2) is given by the following expression:⁷⁾

$$G \equiv \begin{pmatrix} 0 & I \\ -\Sigma & 0 \end{pmatrix} ; \quad \Sigma \equiv \left(\frac{\partial^2 V}{\partial q_i \partial q_j} \Big|_{\vec{q}_0} \right). \quad (2.3)$$

Here I is the $N \times N$ identity matrix. Eigenvalues of the stability matrix G are functions of configuration space point \vec{q}_0 . They determine evolution of small deviations between two neighbor trajectories in the phase space. Namely solution of linearized Hamilton equations valid in the vicinity of considered configuration space point has the form:

$$\begin{pmatrix} \delta\vec{q}(t) \\ \delta\vec{p}(t) \end{pmatrix} = \sum_{i=1}^{2N} C_i \exp\{\lambda_i t\} \begin{pmatrix} \delta\vec{q}(0) \\ \delta\vec{p}(0) \end{pmatrix}. \quad (2.4)$$

Here $\lambda_i = \lambda_i(\vec{q}_0)$ are eigenvalues of the stability matrix G . And $\{C_i\}$ is a full set of projectors. From (2.4) it is seen:

a) If there is i such as $\text{Re } \lambda_i > 0$ then the distance between neighboring trajectories grows exponentially with time and motion is locally unstable. According Liouville's theorem stretching of phase space flow in one direction ($\text{Re } \lambda_i > 0$) is accompanied by its compression in other direction (directions) in order to keep phase space volume constant. That means the existence of $\text{Re } \lambda_j < 0$. Thus for local instability of motion we can demand existence of $\text{Re } \lambda_k \neq 0$.

b) If for any $i = \overline{1, 2N}$ $\text{Re } \lambda_i = 0$ then there is no local instability and the motion is regular.

In the case of *constant* increments of local instability $\{\lambda_i\}$ two-point connected Green function (2.1) in the semiclassical limit can be represented in the form:⁷⁾

$$G_i(t_1, t_2) = \frac{i}{2} \text{Re} \left(\frac{e^{-\lambda_i(t_1-t_2)}}{\lambda_i} \right), \quad t_1 > t_2. \quad (2.5)$$

From the expression (2.5) it is seen:

a) If classical motion is locally unstable (chaotic) then according criterion stated above there is real eigenvalue λ_i . Therefore Green function (2.5) exponentially goes to zero for some i when $(t_1 - t_2) \rightarrow +\infty$. Opposite is also true. If Green function (2.5) exponentially goes to zero under the condition $(t_1 - t_2) \rightarrow +\infty$ for some i , then there exists real eigenvalue of the stability matrix and thus classical motion is locally unstable.

b) If all eigenvalues of the stability matrix G are pure imaginary, that corresponds classically stable motion, then in the limit $(t_1 - t_2) \rightarrow +\infty$ Green function (2.5) oscillates as a sine. Opposite is also true. If for any i Green functions oscillate in the limit $(t_1 - t_2) \rightarrow +\infty$ then $\{\lambda_i\}$ are pure imaginary for any i and classical motion is stable and regular.

Thus we have demonstrated for any finite number of degrees of freedom that proposed quantum chaos criterion coincides with classical criterion of local instability in the semiclassical limit of quantum mechanics in the case when increments of local instability are constant (corresponding principle).

In the case of non-constant λ s the calculation of two-point connected Green function in the whole range of variation of its arguments is several orders more complicated problem then in the case of constant ones. The condition for Green function to be finite in the limit of infinite distance between its arguments forced us to eliminate exponentially growing item from the expression (2.5). However, it is not so in general case, when we can consider increments of instability as constants just in small region around the considered point of configuration space. Therefore we can not demand the elimination of exponentially growing item and the expression for two-point connected Green function valid in sufficiently small region of configuration phase space is:

$$G_i(t_1, t_2) = D_1^{(i)} e^{\lambda_i(t_1 - t_2)} + D_2^{(i)} e^{-\lambda_i(t_1 - t_2)}, \quad (2.6)$$

where $D_1^{(i)}$, $D_2^{(i)}$ are arbitrary constants and $t_1 - t_2$ is assumed to be sufficiently small. Thus for non-constant λ s we can describe *local* behavior of Green function, but we are not able to predict its *global* behavior that is needed for proposed chaos criterion to be applied.

Above we provided justification of proposed chaos criterion for non-relativistic quantum systems with any finite number of degrees of freedom which can be considered as QFT in $0 + 1$ dimensions. However quantum mechanics is just formally similar to quantum field theory. Physically they are different systems. Namely, quantum mechanical systems considered above have any but *finite* number of degrees of freedom whereas quantum fields possess *infinite* number of freedoms and have to be considered in the space-time which dimension is larger then one. Rapid decreasing of the Green function (2.1) for quantum mechanical system means localization in configuration space *) of the system while for quantum fields localization in space-time is required. Why did we consider quantum mechanics in this case? The answer is that for field system localization in its configuration space is the necessary condition for the space-time localization. This provides us the way for further justification of the proposed chaos criterion.

Now we qualitatively justify the correspondence between proposed quantum chaos criterion in semiclassical limit and chaotic behavior of classical fields in the general case of arbitrary field system. Some specification are possible in concrete cases. But we believe that the sketch of prove remains the same.

We state that chaotic behavior of classical fields leads to exponential or faster decreasing of the Green function (2.1) in the semiclassical limit. Without loss of

*) The notion of configuration space can be introduced since we consider semiclassical limit.

generality we can consider the system consisting from the fields and/or field components $\varphi_a(x)$. Here x denotes the space-time 4-vector, a enumerate fields and field components. Let us consider the field system on a spatial lattice and leave time continuous. We break relativistic invariance in the manner widely used in the lattice QFT.²⁶⁾ Field functions $\varphi_a(\vec{x}_j, t)$ in discrete space are generalized coordinates of the field system in the classical case and play the role of Heisenberg generalized coordinate operators after the secondary quantization. They obey usual quantum mechanical commutation relations:

$$[\varphi_a(\vec{x}_i, t), \varphi_b(\vec{x}'_j, t)] = i\hbar_{eff}\delta_{ab}\delta_{ij}. \quad (2.7)$$

Here \hbar_{eff} is the effective Plank constant. Hamiltonian of the field system on a spatial lattice has the form (2.2). Chaotic behavior of classical fields means uncorrelated behavior in different space points and exponential or even faster decay of the classical correlation function with time. That implies locally unstable motion in the phase space of the field system. According the results of Ref.⁷⁾ it leads to exponential decreasing of the Green function (2.1) for constant eigenvalues of the stability matrix (increments of local instability). The general case of non-constant increments of local instability can be considered only qualitatively. There are indications originated from quantum mechanics that localization in the phase space appears in the presence of classical chaos, however the question about the relation between classical chaos and quantum dynamical localization is still open.¹⁹⁾

Qualitative justification of proposed chaos criterion in the general case looks as follows. Field system on a lattice built above is a Hamiltonian system with large number of degrees of freedom and bounded motion in its phase space (corresponding boundary conditions are assumed). In semiclassical regime which is considered there exists break (or Heisenberg) time τ_H . It determines the time scale up to which quantum dynamics follows classical one. Break time decreases when effective Plank constant \hbar_{eff} determined by (2.7) increases. If Heisenberg time becomes less then the classical diffusion time τ_D needed the system to cover all available phase space ($\tau_H < \tau_D$) then the field system is dynamically localized in the phase (therefore configuration) space (see the Ref.¹⁹⁾). That is the necessary condition for the space-time localization required by the proposed chaos criterion.

To demonstrate it consider localized in space field configuration. In the simplest case localization means the field functions to be zero outside some finite hypersphere in configuration space of the system. As well fields are assumed to take finite values because of physical reasons (no singularities). Therefore, under the conditions stated the space-time localization leads to the localization in the configuration space. That was needed to prove. The question about the sufficient conditions is still open.

§3. Consequences of proposed chaos criterion

In this section we discuss consequences of deterministic chaos in QFT following from the proposed chaos criterion (see the Sec.2). Particulary, rapid (exponential or faster) decrease of the propagator (2.1) implies the system to be confined in some region of the space-time. The same behavior of the propagator is required in order to

provide the regime of superlocalization²⁷⁾ needed for explanation of the confinement of colored objects (quarks and gluons) in QCD.²⁸⁾ Therefore deterministic chaos in QFT regarded in the framework of our approach can be considered as the sufficient condition for the confinement phenomenon to occur. Moreover the direct connection between the confinement of particles and stochastic behavior of the background classical fields was obtained.²⁸⁾ This fact give additional argument *) in favour of the chaos criterion proposed in the Ref.⁷⁾ The condition of stochasticity of classical background non-abelian gauge fields in the simplest case means fast enough decrease of the bilocal correlator in the cluster expansion²⁹⁾ of the path ordered exponential in the fundamental representation³⁰⁾ (conditions needed for applicability of non-abelian Stocks theorem³⁰⁾ are assumed to be justified). Roughly, for confinement of quarks and gluons to occur (with linear confining potential in non-relativistic limit) existence of the finite correlation length for the classical background non-abelian gauge fields is needed.²⁸⁾ In the Ref.²⁸⁾ the stochastic assembly of background fields imposed "by hands" was considered. We note that it can be realized by the classical chaotic solutions of the field equations of motion. These configurations are found both as exact solutions of gauge field equations^{8),10)} (see references wherein) and in lattice simulations.^{9),16)} They are not vacuum configurations and the condition of (anti-) self-duality assumed in Refs.^{28),30)} is broken. However they realize the local minimum of the field action and therefore provide non-zero amplitude for realization of the confinement mechanism. Moreover due to a large number of such solutions their contribution can be essential and even exceed the contribution of vacuum configurations, this is the question for further investigations.

One of possible applications of proposed chaos criterion in field theory is an investigation of the stability of classical solutions with respect to small perturbations of initial conditions. Of course, this does not directly imply chaos, but advances us to it. To study the stability of certain classical solution of field equations one has to calculate (for instance, using one loop approximation) two-point Green function in the vicinity of considered classical solution.

To demonstrate this, consider real scalar φ^4 -field:

$$L = \frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4!} \varphi^4. \quad (3.1)$$

Here $\lambda > 0$ is a coupling constant, m^2 is some parameter which can be larger or less then zero. In both cases $\varphi = 0$ is a solution of field equations. Asymptotic of two-point Green function calculated in the vicinity of the classical solution $\varphi = 0$ in the zero order of perturbation theory is:

$$G(x, y) \underset{\rho \rightarrow \infty}{\sim} \rho^{-\frac{1}{2}} e^{im\sqrt{\rho}}. \quad (3.2)$$

Here $\rho = (x - y)^2$ and we accept that 4-vector $x - y$ is inside the light cone $(x^0 - y^0) > 0$, in other words $\rho > 0$. We can study the stability of considered solution with respect to small perturbations. Expression (3.2) shows that we have two different cases:

*) It does not prove the correspondence between proposed quantum chaos criterion and classical ones, because the deterministic chaos is the sufficient condition for stochastic behavior but it is not the necessary one.

a) Green function oscillates and slowly (non-exponentially) goes to zero when $\rho \rightarrow \infty$. According proposed chaos criterion considered solution is stable. Indeed, from (3.2) it follows that parameter m is real in this case. Therefore $\varphi = 0$ is a stable vacuum state.

b) Green function exponentially goes to zero in the limit $\rho \rightarrow \infty$. From proposed chaos criterion it follows that $\varphi = 0$ is an unstable solution. That is true since from (3.2) one can see that parameter m has to be pure imaginary. It is known that in this case state $\varphi = 0$ becomes unstable, two new stable vacuums are appeared and we obtain spontaneous symmetry breakdown.³¹⁾

Thus for real scalar φ^4 -field spontaneous symmetry breakdown and degeneration of vacuum state can be regarded as signatures of quantum chaos. This relates our approach with the symmetry approach of Bunakov^{6),21)} (see discussion in the Sec. 1). Namely, on the particular example we demonstrated proposed chaos criterion to lead to the ground state symmetry violation.

§4. Conclusion

In this work we have demonstrated the necessity and substantiated the validity of introduction the notion of deterministic chaos in QFT. We briefly reviewed existing approaches to this problem. We continued the justification of the chaos criterion for quantum fields proposed by us in our earlier papers.^{7),10)} Particulary, we demonstrated semi-qualitatively that exponential (or faster) decreasing of the two-point connected Green function (2.1) is the sufficient condition for chaotic behavior of the fields in the classical limit. Our qualitative arguments are supported by the results obtained in Refs.^{27),28),30)} in connection with confinement problem in QCD. We also conjectured that confinement of quarks and gluons can be provided by classical chaotic solutions of Yang-Mills equations. Relation between chaos in QFT systems and their symmetry violation has been discussed on a particular example of the $\lambda\varphi^4$ field system.

We gave a brief review of the problems standing on the way to the understanding the nature and consequences of deterministic chaos in QFT. Particulary, the further justification of the proposed chaos criterion is needed, as well its consequences has to be clarified. Connection with the longstanding problems of particle physics, such as confinement problem in QCD and other non-perturbative phenomena, has to urge forward the investigation of deterministic chaos in QFT.

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